Stetig / diffbar: HS11 - 2.2 a)

$$f(x) = \begin{cases} \alpha + x & x \leq 0 \\ \beta e^x & x > 0 \end{cases} \qquad \alpha, \beta \in \mathbb{R}$$

a) $\lim\limits_{x \uparrow 0^-} f(x) \overset{!}{=} \lim\limits_{x \downarrow 0^+} f(x)$ <u>und</u> $\lim\limits_{x \uparrow 0^-} f'(x) = \lim\limits_{x \downarrow 0^+} f'(x)$

$$\alpha + 0 \overset{!}{=} \beta e^0$$

$$\boxed{\alpha = \beta}$$

$$f'(x) = \begin{cases} 1 & x \leq 0 \\ \beta e^x & x > 0 \end{cases}$$

$$\Rightarrow \quad 1 \overset{!}{=} \beta e^0$$

$$\boxed{1 = \beta}$$

$$\Rightarrow \underline{\underline{\alpha = \beta = 1}}$$

$f(x)$ auf $\mathbb{R}$ diff'bar für $(\alpha, \beta) = (1, 1)$

Pr07 - Aufgabe 2

$$f(x) = \begin{cases} Ae^x & \text{falls } x < 0 \\ B\frac{1}{1+x^2} & \text{falls } x \geq 0 \end{cases} \qquad x_0 = 0$$

a)

$$\lim_{x \uparrow x_0} f(x) = \lim_{x \downarrow x_0} f(x)$$

$$\lim_{x \uparrow 0} Ae^x = \lim_{x \downarrow 0} B\frac{1}{1+x^2}$$

$$A = Ae^0 = B \cdot 1 = B \quad \Rightarrow \boxed{A = B}$$

b)

① stetig $\rightarrow A = B \Rightarrow f(x) = \begin{cases} Ae^x & x < 0 \\ A\frac{1}{1+x^2} & x \geq 0 \end{cases}$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$= \int_{-\infty}^{0} Ae^x\,dx + \int_0^{\infty} A\frac{1}{1+x^2}\,dx = A\int_{-\infty}^{0} e^x\,dx + A\int_0^{\infty}\frac{1}{1+x^2}\,dx$$

$$= Ae^x \Big|_{-\infty}^{0} + A \cdot \underbrace{\arctan(x)}_{\tan^{-1}(x)} \Big|_0^{\infty}$$

$$= A \cdot e^0 - \text{"}\underbrace{Ae^{-\infty}}_{0}\text{"} + \text{"}A \cdot \underbrace{\tan^{-1}(\infty)}_{\frac{\pi}{2} \text{ (siehe Trigonometrie-Blatt)}}\text{"} - A\underbrace{\tan^{-1}(0)}_{0}$$

$$= A - 0 + A\frac{\pi}{2} - 0$$

$$= A\left(1 + \frac{\pi}{2}\right) \stackrel{!}{=} 1 \quad \Big| : \left(1 + \frac{\pi}{2}\right)$$

$$A = \frac{1}{1+\frac{\pi}{2}} \Rightarrow B = \frac{1}{1+\frac{\pi}{2}} = \frac{1}{\frac{2}{2}+\frac{\pi}{2}} = \frac{1}{\frac{2+\pi}{2}} = 1 \cdot \frac{2}{2+\pi}$$

$$= \frac{2}{2+\pi}$$

PrO7 - A2 Fortsetzung

d) gem. b) gewählte Fn: $f(x) = \begin{cases} \frac{2}{2+\pi} \cdot e^x & x < 0 \\ \frac{2}{2+\pi} \cdot \frac{1}{1+x^2} & x \geq 0 \end{cases}$

$$\Rightarrow f'(x) = \begin{cases} \frac{2}{2+\pi} \cdot e^x & x < 0 \\ \frac{2}{2+\pi} \cdot \frac{0 \cdot (1+x^2) - 1 \cdot 2x}{(1+x^2)^2} = \frac{2}{2+\pi} \cdot \frac{-2x}{(1+x^2)^2} & x \geq 0 \end{cases}$$

differenzierbar: $\lim\limits_{x \uparrow x_0} f(x) = \lim\limits_{x \downarrow x_0} f(x)$

✓ stimmt da gem. b) A=B gewählt wurde

und

$$\lim_{x \uparrow x_0} f'(x) = \lim_{x \downarrow x_0} f'(x)$$

$$\lim_{x \uparrow 0} \frac{2}{2+\pi} e^x = \lim_{x \downarrow 0} \frac{2}{2+\pi} \cdot \frac{-2x}{(1+x^2)^2}$$

$$\frac{2}{2+\pi} e^0 \neq \frac{2}{2+\pi} \cdot \frac{0}{1^2}$$

$$\frac{2}{2+\pi} \neq 0$$

$\Rightarrow$ nein, die Fn ist in $x_0 = 0$ nicht diff'bar!

stetig / diffbar:

HS09 - 1.2 i)

$$f(x) = \begin{cases} x^2 - 2ex + m & x < e \\ x^2 \ln(x) & x \geq e \end{cases}$$

$$\lim_{x \uparrow e^-} f(x) \overset{!}{=} \lim_{x \downarrow e^+} f(x)$$

$$e^2 - 2e^2 + m = e^2 \cdot \underbrace{\ln(e^1)}_{=1}$$

$$-e^2 + m = e^2 \qquad | +e^2$$

$$\underline{\underline{m = 2e^2}}$$

stetig/diffbar:

HS10 - 1.2

$$g(x) = \begin{cases} ax^3 & x \le 1 \\ x^2 + b & x > 1 \end{cases} \qquad a, b \in \mathbb{R}$$

i) $\lim_{x \uparrow 1-} g(x) \stackrel{!}{=} \lim_{x \downarrow 1+} g(x)$

$a \cdot 1^3 = 1^2 + b$

$\boxed{a = 1 + \underline{b}} \Rightarrow (a, b) = (1+b, b) \quad b \in \mathbb{R}$

resp. $\{(a,b) \in \mathbb{R}^2 \mid a = 1 + b\}$

ii) $\lim_{x \uparrow 1-} g(x) \stackrel{!}{=} \lim_{x \downarrow 1-} g(x)$ <u>und</u> $\lim_{x \uparrow 1-} g'(x) \stackrel{!}{=} \lim_{x \downarrow 1-} g'(x)$

$\boxed{a = 1 + b}$

$$g'(x) = \begin{cases} 3ax^2 & x \le 1 \\ 2x & x > 1 \end{cases}$$

$\Rightarrow 3a1^2 = 2 \cdot 1$

$3a = 2 \quad | :3$

$\frac{2}{3} = 1 + b \mid -1 \quad \leftarrow \quad \boxed{a = 2/3}$

$-\frac{1}{3} = b$

$\Rightarrow (a, b) = \left(\frac{2}{3}, -\frac{1}{3}\right)$

iii) $\lim_{x \uparrow 1-} g''(x) \stackrel{!}{=} \lim_{x \downarrow 1-} g''(x)$ <u>und</u> $\underbrace{\lim_{x \uparrow 1-} g'(x) \stackrel{!}{=} \lim_{x \downarrow 1+} g'(x) \text{ und } \lim_{x \uparrow 1-} g(x) \stackrel{!}{=} \lim_{x \downarrow 1+} g(x)}$

$\circledast \quad 4 = 2 \quad ↯ \qquad a = 2/3 \quad b = -\frac{1}{3} \Rightarrow g'(x) = \begin{cases} 2x^2 & x \le 1 \\ 2x & x > 1 \end{cases}$

$\Rightarrow$ g ist <u>nicht</u> zweimal differenzierbar

$\circledast \Leftarrow g''(x) = \begin{cases} 4x & x \le 1 \\ 2 & x > 1 \end{cases}$